



Smith, DA., Marklof, J., & Wilson, RE. (2008). *Improved power law potentials for highway traffic flow*. <http://hdl.handle.net/1983/1061>

Early version, also known as pre-print

[Link to publication record in Explore Bristol Research](#)
PDF-document

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
<http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/>

Improved power law potentials for highway traffic flow

David A. Smith¹, Jens Marklof¹, and R. Eddie Wilson²

¹ Department of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, United Kingdom

² Bristol Centre for Applied Nonlinear Mathematics, Department of Engineering Mathematics, University of Bristol, Queen's Building, University Walk, Bristol BS8 1TR, United Kingdom

Received: date / Revised version: date

Abstract. The established relationship between the spacing distributions of highway traffic and those of classical gas models is re-examined. Limitations in the statistics of the power law potential are identified, and two modifications are proposed which alter the potential at respectively long and short ranges. An improved fit with empirical data is demonstrated.

PACS. 05.10.Gg Stochastic analysis methods – 05.20.Gg Classical ensemble theory – 89.40.-a Transportation

1 Introduction

Studies [1–5] in recent years have examined the distributions of spacings in highway traffic, and have drawn links to distributions from classical statistical physics models. The spacing distribution can be related to the interaction potential $V(s)$ via methods from equilibrium statistical physics [6,7], and this potential can be used to inform microscopic traffic models. In a microscopic driven-agent model we can describe the motion of an agent with the general car-following (Langevin) equation [4,8]

$$\frac{dv_i}{dt} = \frac{v_{\max} - v_i}{\tau} + f(s_i) - \gamma f(s_{i-1}) + \eta_i(t), \quad (1)$$

where v_i is the speed of agent i , $s_i = x_i - x_{i+1}$ is the gap to agent $i + 1$ and $f(s) := -V'(s)$ denotes the interaction force with the neighbouring agent. Finally $\gamma = 0$ gives the case of forwardly directed interactions, whilst $\gamma = 1$ gives symmetric (forward and backward) interactions, τ is an adaptation time, and $\eta_i(t)$ is a noise term.

In this short note we extend the work of Krbálek and Helbing [4,5] who considered power law potentials $V(s) = s^{-\alpha}$, where α is found by the best fit between the consequent theoretical spacing distribution and empirical traffic spacings. By analysing the fit with our data source (provided by inductance loops from London's M25 orbital mo-

torway), we find discrepancies with Krbálek and Helbing's results. We then exhibit two improvements to the power law potential which include respectively an exponential factor at large distances and a short range attractive region.

2 Power law potential: comparison with data

We follow [7] and consider a one-dimensional gas of N particles confined to a ring lattice of length L . Particles interact through a repulsive two-body potential $V(s)$ which extends so that each particle can interact with its nearest neighbours (on each side). The nearest neighbour spacing distribution $P(s)$, describing the probability of finding a distance s between two neighbouring particles, is then given by [5, 7]

$$P(s) = Ae^{-cs}e^{-\beta V(s)}. \quad (2)$$

We now choose $V(s) = s^{-\alpha}$ and consider the fit of $P(s)$ against locally-averaged empirical spacing data as α is varied.

The data we use is from the UK Highways Agency's MIDAS system on the M25 orbital motorway around London. This system includes inductance loops embedded in the road surface, which in standard operation collect one minute averages of traffic flow statistics, such as flow, velocity, occupancy etc. However, adjustments to the roadside hardware allow the collection of a full (unaveraged) individual vehicle data (IVD) set. Here we use data from one inductance loop site, and after cleaning the data of anomalous (unphysical) records, we estimate the spacing

of two consecutive vehicles by their time difference (i.e., their time headway) multiplied by the velocity of the following vehicle. In low density regimes, vehicle velocities may be highly uncorrelated, in which case refinements of this approach should be considered — a point which is beyond the scope of this paper.

Since the parameter α is known to depend on the flow regime (i.e., whether traffic is congested, or free flow etc.), we take local mean spacings of 50 vehicles up- and downstream of each vehicle, from which we compute the local density. The constants A and c are then fixed through normalisation,

$$\int_0^\infty A \exp\left(-\frac{\beta}{s^\alpha} - cs\right) ds = 1, \quad (3)$$

and by rescaling the data via

$$\int_0^\infty As \exp\left(-\frac{\beta}{s^\alpha} - cs\right) ds = 1, \quad (4)$$

so that the mean spacing is one. For comparison with data, the parameter β is chosen for best fit using the least-squares method applied to the cumulative spacing distribution, since this eliminates bin-size effects which can occur with histogram data.

Following this procedure, Krbálek and Helbing [4] found $\alpha = 4$ to give the best agreement with data for free flowing traffic (density ≤ 20 veh./km/lane), but found $\alpha = 1$ to be best for high density traffic. However, Krbálek in a later study [5] found $\alpha = 1$ to give the best agreement over all densities.

In contrast, we have found that no single value of α gives significantly better agreement with the data than others over a range of densities, but $\alpha = 1$ is notably

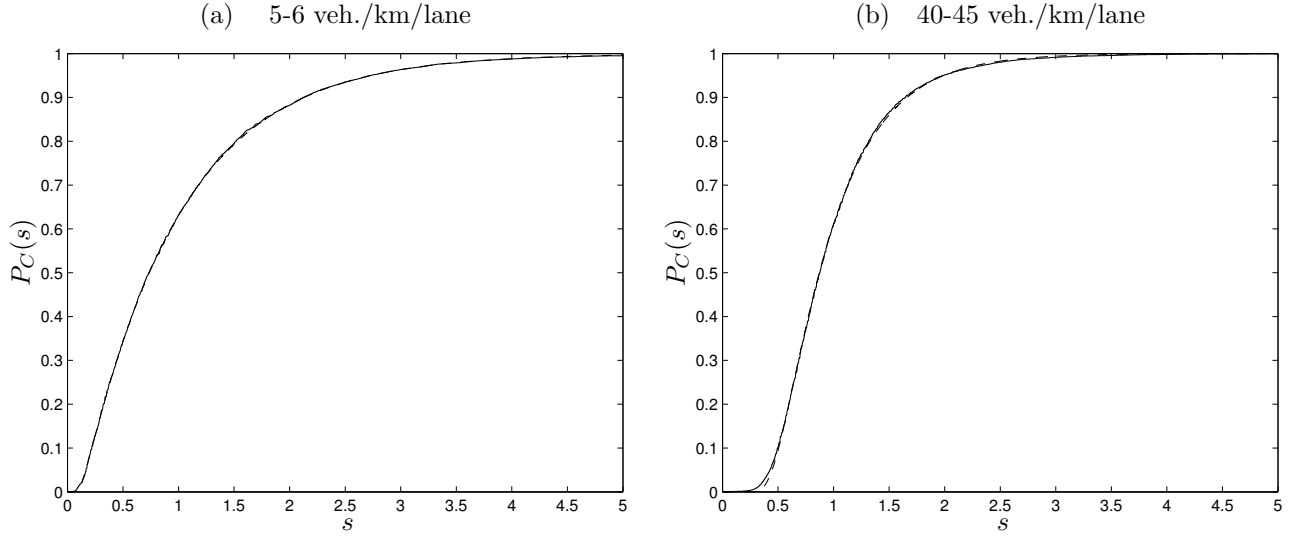


Fig. 2. Cumulative spacing distribution $P_C(s)$ for power law potential with $\alpha = 3$ (---). The parameter β has been fitted by comparison with road traffic data (—). The two density regimes shown here are representative of fits over the full range, and we can see broadly the change in the shape of the distribution with increased density. Because the empirical and theoretical plots are so similar we shall henceforth just show difference plots, as in Figure 3.

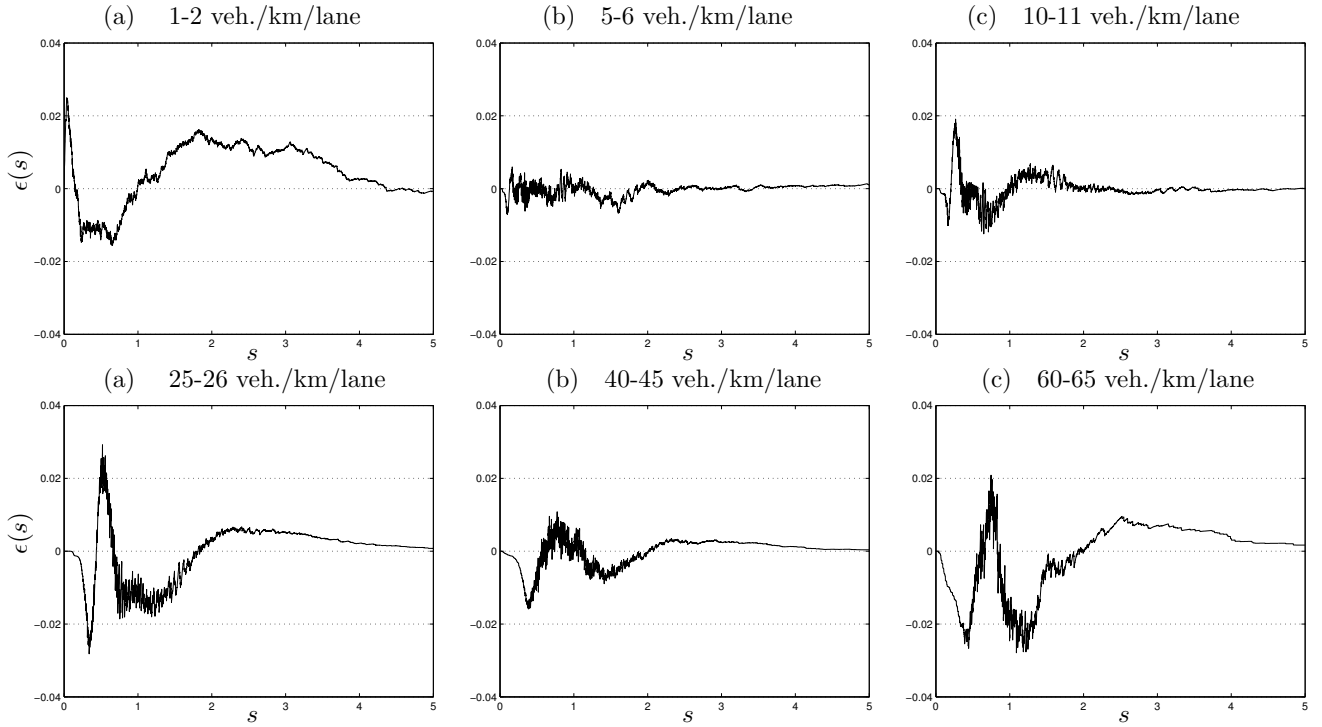


Fig. 3. Error plots for the cumulative spacing distribution for the power law potential with $\alpha = 3$, showing the discrepancy between the theoretical model and empirical data.

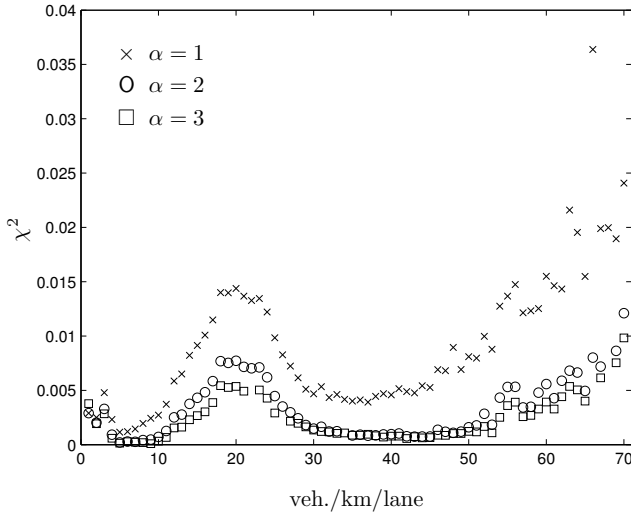


Fig. 1. Deviation χ^2 between spacing distributions from the gas model with power law potential and empirical traffic data. The $\alpha = 3$ case (as plotted in Figure 2) is shown for comparison with $\alpha = 1, 2$.

worse than other values. In Figure 1 we see this demonstrated by comparison of the χ^2 deviation from the data for $\alpha=1, 2$ and 3. The best agreement appears to be for $\alpha = 3$, especially at densities around 40 veh./km/lane. Comparisons of the cumulative spacing distributions with data are shown for $\alpha = 3$ in Figures 2 and 3.

In Figure 4 we show the fitted values of the inverse temperature β plotted against density, for $\alpha = 3$. We see a broadly monotonic increase with density, which seems intuitive but is in contrast with the result for $\alpha = 1$ in [5], where β was seen to decrease between 35 and 50 veh./km/lane before increasing again. A possible reason for our different findings is that [5] removes records corresponding to trucks before analysis, whereas we include all vehicles.

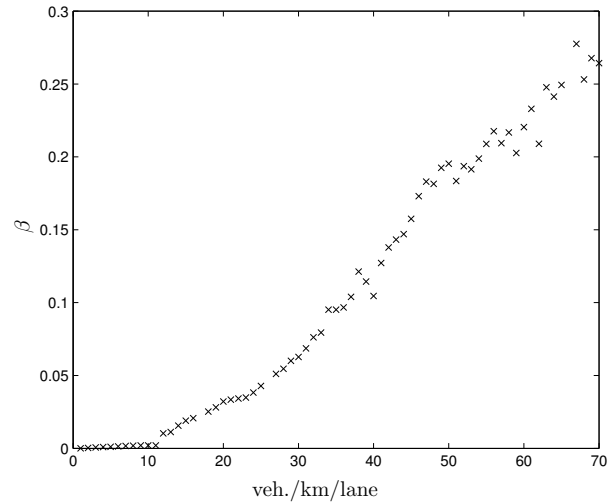


Fig. 4. Values of β calculated to fit empirical data over a range of density values for $\alpha = 3$.

3 Refined power law potentials

To further examine the fit to data obtained from the power law potential at different α values, we plot the spacing distributions on a logarithmic scale for two different traffic densities in Figure 5. This enables us to see how well the models capture the distribution in the tail. In accordance with our findings from comparison of the χ^2 deviations, we see the worst fit for $\alpha = 1$ (particularly for higher density), and we find improvement for $\alpha = 2$ and 3, with little further improvement for $\alpha > 3$. These results suggest that $\alpha = 1$ does not have a quick enough drop-off in the potential — something that improves with higher α . However, the improvement at large spacings for higher α is weighed up against a poorer agreement with data at low spacings, which is indicative of the high- α potentials being too repulsive at short range. Also, since the fit to the tail does not change significantly for $\alpha > 3$, it seems there is negligible interaction beyond a critical spacing —

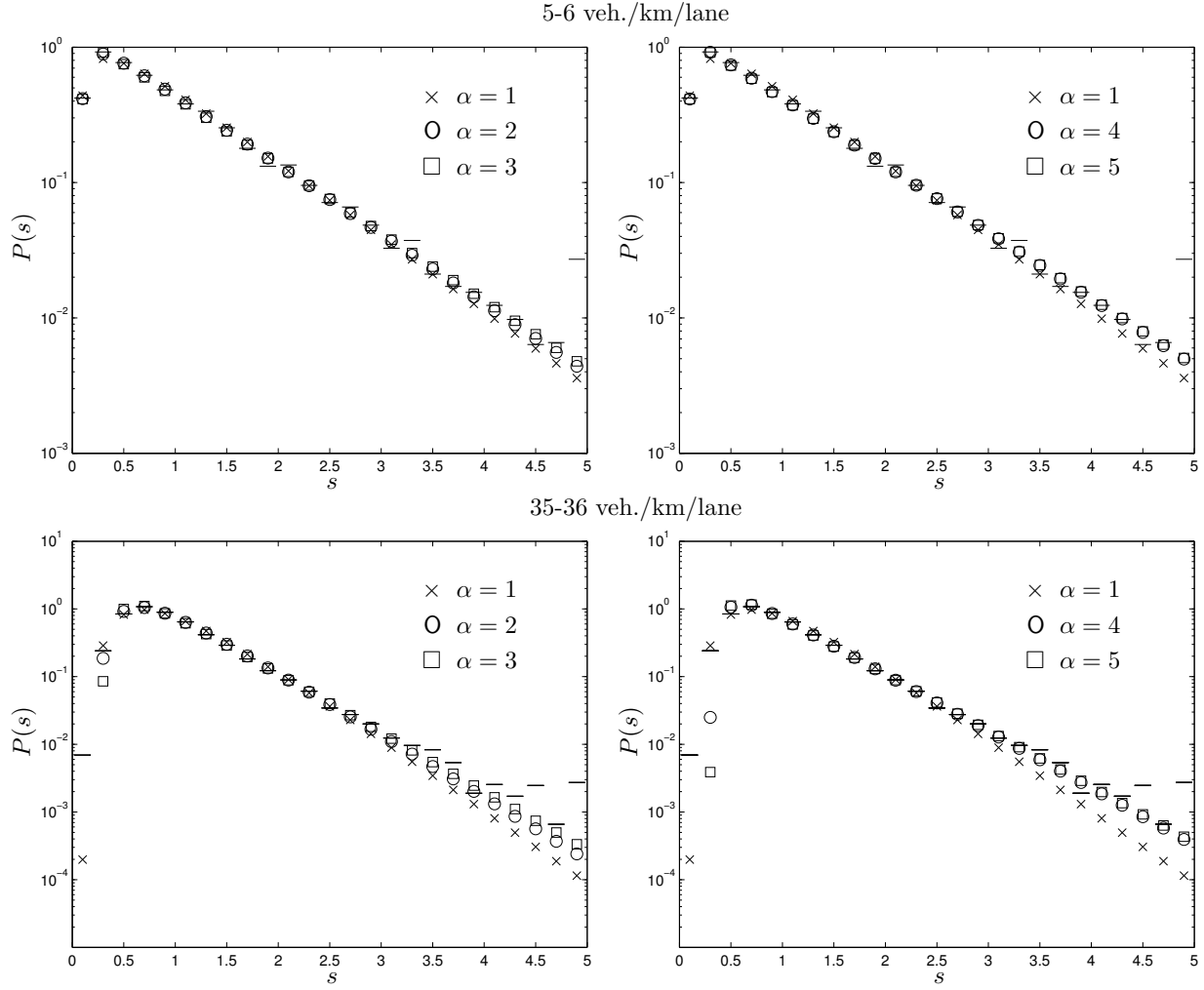


Fig. 5. Spacing distribution $P(s)$ for the power law potential with different values of α . We show a logarithmic scale to examine the fit to data (bars) in the tail of the distribution, and we see that the fit improves for larger α .

so perhaps a better solution would be a curtailed $\alpha = 1$ potential.

3.1 Exponential cut-off

We can achieve a curtailed potential either by setting a maximum range of interaction, or by multiplying the power law potential by a rapidly falling function, such as an exponential, giving $V(s) = e^{-\gamma s} s^{-\alpha}$.

In Figure 6 we display the results for an exponentially curtailed power law potential with $\alpha = 1$, for which the spacing distribution (2) is given by

$$P(s) = A e^{-cs} e^{\beta(s^{-1} e^{-\gamma s})}, \quad (5)$$

which is now fitted to data by varying both β and γ . We see smaller differences between the model and the data than for the pure power law potential with $\alpha = 3$. In Figure 7 we show the fitted values of β and γ across the full range of densities. The decay parameter γ remains

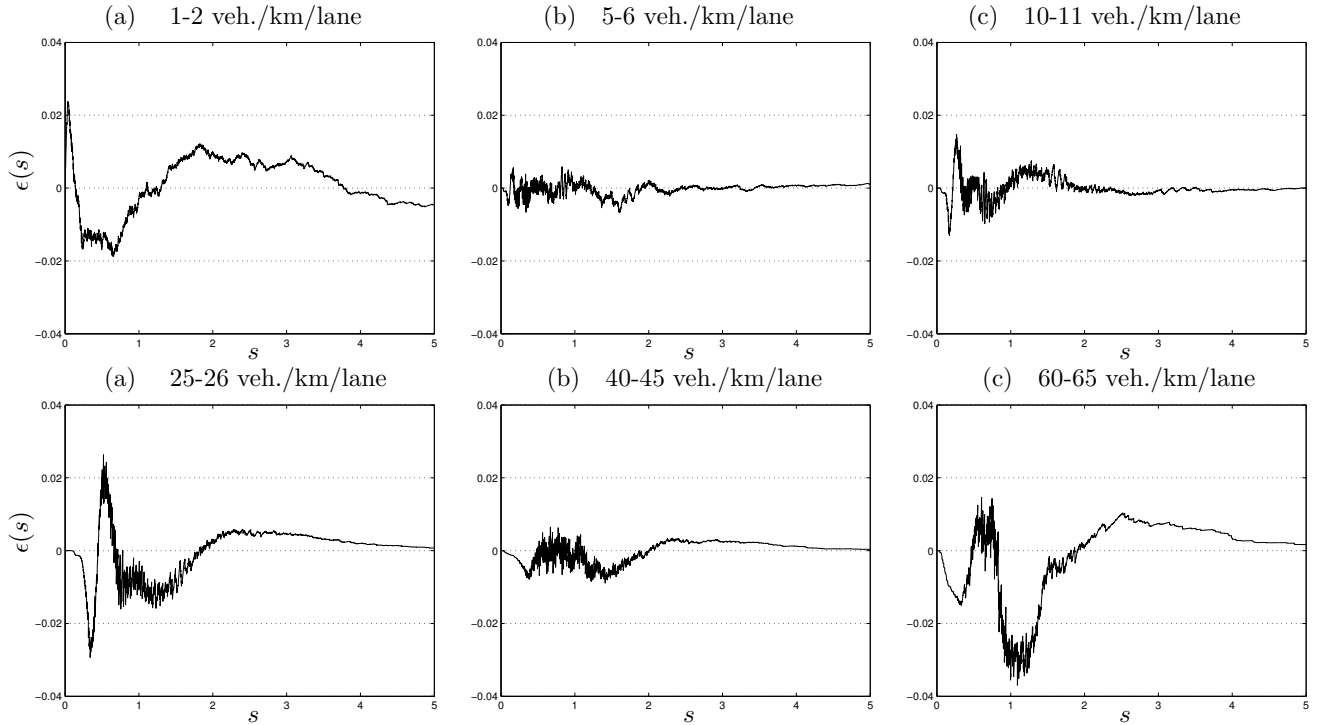


Fig. 6. Error plots for the cumulative spacing distribution for power law potential with exponential curtailment, showing the discrepancy between the theoretical model and empirical data.

above 3, even at high densities, giving a sharp drop-off in the potential — at a third of the mean separation the interaction potential has fallen by at least half.

3.2 Short range attraction

The exponential cut-off model assumes the low order ($\alpha = 1$) power law potential to be a good model for closely-spaced vehicles and seeks to correct the poor fit at larger spacings. Another interpretation of the comparison between empirical data and power law potentials is that we should consider α values which fit to the tail of the distribution, and then modify the potential at short range.

We propose a form of the interaction potential which has strong power law decay combined with a region of

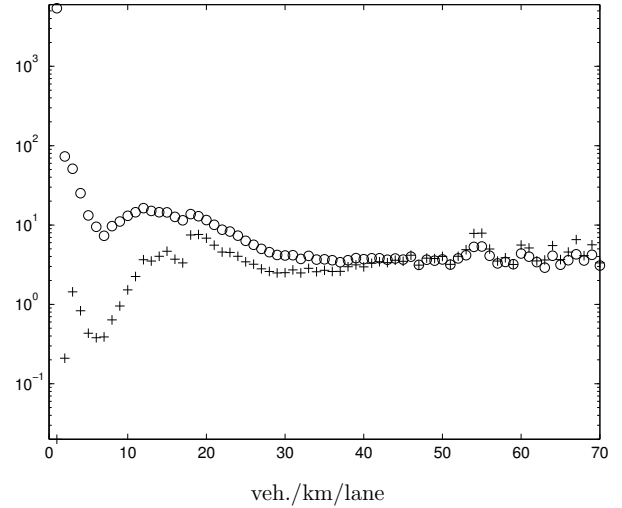


Fig. 7. Values of β (+) and γ (o) from fitting the potential $V(s) = e^{-\gamma s} s^{-1}$ to empirical data. For very low density we see large γ and small β , effectively switching off the interaction. For higher densities (> 30 veh./km/lane) γ takes values in the range 3 to 5.

attraction at close range, namely

$$V(s) = \frac{(1 - as)^2}{s^b}. \quad (6)$$

This form maintains ‘hardcore’ repulsion at very short range (to avoid collisions), so that the attraction at longer range tends to lead to regularly spaced platoons.

With choice (6) we have

$$P(s) = Ae^{-cs}e^{\beta(1-as)^2/s^b}, \quad (7)$$

where, as before, A and c are fixed by the normalisation condition (3) and by setting the mean spacing to one (4). This leaves the parameters β , a and b free for fitting. Figure 8 shows difference plots of the cumulative distribution and empirical data at various densities, whereas Figure 9 shows the χ^2 deviation between the model and data over a range of densities and also shows comparison with the potentials we considered earlier. In general, the short range attraction model is best, especially in the density range 10 and 30 veh./km/lane. Of course, an extra free parameter was varied here and so one should expect a better fit. Thus further work should focus on whether the improvement is statistically significant.

4 Conclusion

We have shown that the spacing statistics for the short-range gas model with a simple power law interaction potential are similar to those recorded in empirical road traffic data, in agreement with previous work by Krbálek and Helbing [4,5]. However, the form of the potential can be modified to improve the fit with empirical data. We sug-

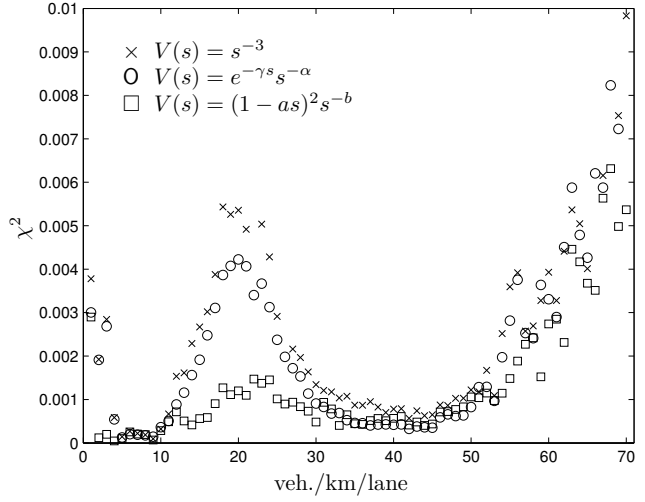


Fig. 9. Comparison of χ^2 deviations between traffic data and the interaction potential given by the power law with $\alpha = 3$ (\times), with exponential curtailment (\circ), and with short range attraction (\square).

gest that a ‘traffic-like’ potential should drop off rapidly with distance and include short range attraction.

R.E. Wilson acknowledges the support of an EPSRC Advanced Research Fellowship EP/E055567/1.

References

1. M. Krbálek and P. Šeba. *J. Phys. A*, 33:L229, 2000.
2. M. Krbálek and P. Šeba. *J. Phys. A*, 36:L7, 2003.
3. M. Krbálek, P. Šeba, and P. Wagner. *Phys. Rev. E*, 64:066119, 2001.
4. M. Krbálek and D. Helbing. *Physica A*, 333:370, 2004.
5. M. Krbálek. *J. Phys. A: Math. Theor.*, 40(22):5813, 2007.
6. F. Gürsey. *Proc. Cambridge Phil. Soc.*, 46:182, 1950.
7. E. Bogomolny, U. Gerland, and C. Schmit. *Eur. Phys. J. B*, 19:121, 2001.
8. D. Helbing and M. Treiber. 2006. arXiv:cond-mat/0307219.

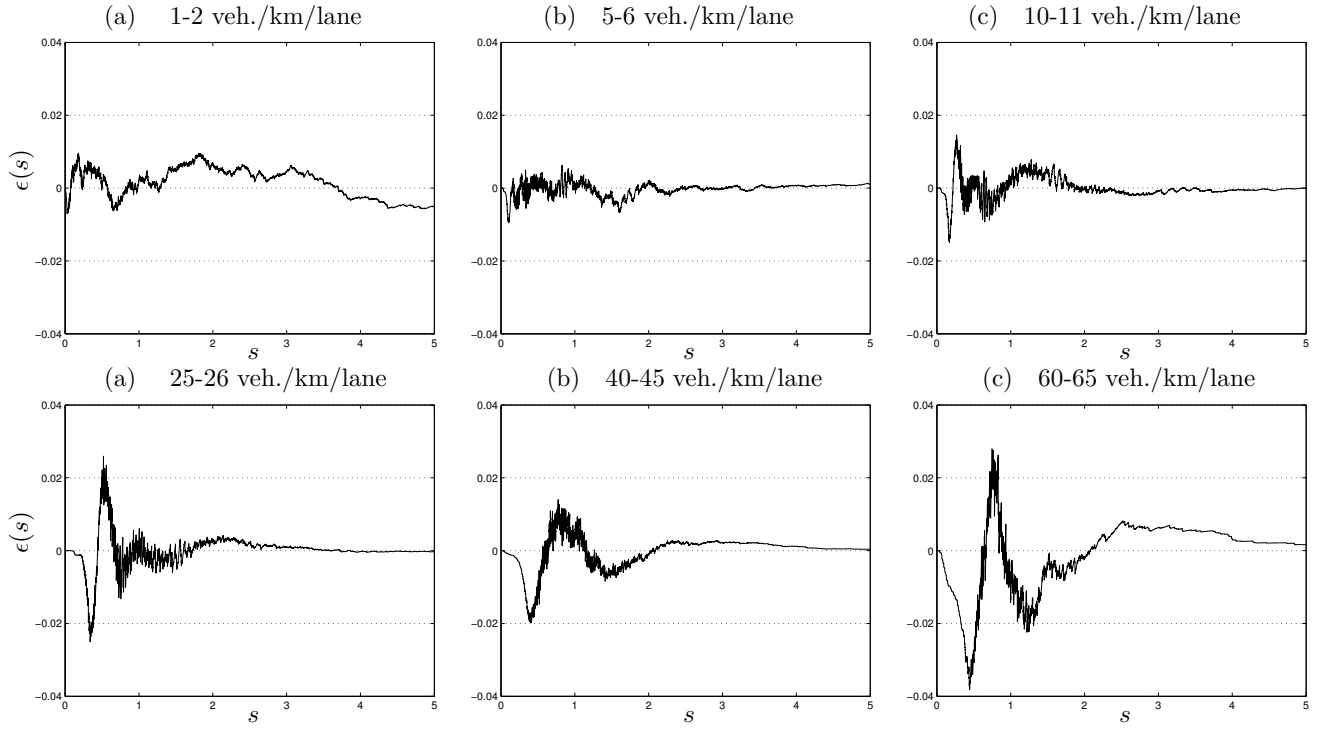


Fig. 8. Error plots for the cumulative spacing distribution for the interaction potential given by (6), showing the discrepancy between the theoretical model and empirical data.